

# Proposal of Surface-Wave Planar Circuit, Formulation of its Planar Circuit equations and its Practical Application

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## Abstract

This paper proposes the concept of a surface-wave planar circuit and formulates surface-wave planar circuit equations based on Maxwell's equations. As examples of the application of these equations to practical problems, step discontinuity, planar-type transmission line and circular-shaped resonator of covered image-line type are considered.

## Introduction

Surface-wave devices are frequently used in the millimetric and optical frequency range as functional devices such as filter, resonator, hybrid, directional coupler, mode filter, mode converter, and so on. The mechanism of these devices mainly depends on the propagation characteristics of the surface-wave in one direction. Therefore, the analysis and synthesis of these devices is fundamentally based on the one-dimensional or transmission-line theory, which is well-established now. However, if we look at these devices in more detail, it turns out that these devices are two dimensional in extent, and the surface-wave will propagate or diverge in two dimensions. Therefore, these devices are better treated as two-dimensional or planar circuit. This is one reason for proposing the concept of surface-wave planar circuit. Another reason is that like the conventional planar circuit, the surface-wave planar circuit has two degrees of freedom in dimension rather than one as in the transmission-line circuit. This gives the planar circuit the possibility of realizing functional devices that are not possible with the transmission-line circuit.

General planar circuit theory is now under development<sup>(1)(2)(3)</sup>. Therefore, if we can derive and formulate planar circuit equations for a surface wave, we can analyze and/or synthesize these devices more exactly and systematically through planar circuit theory.

Keeping the above points in mind, the following items are treated in this paper.

1. the concept and structure of the surface-wave planar circuit.
2. the formulation of surface-wave planar circuit equations based on Maxwell's equations.
3. the application of surface-wave planar circuit equations of covered image-line type to practical problems.

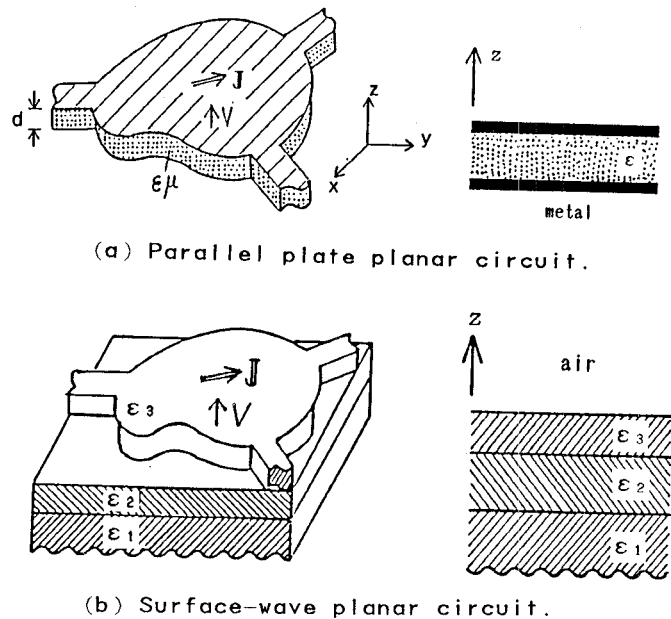


Fig.1 Planar circuit.

## Concept and Structure

A planar circuit must have a mechanism or structure which confines the electromagnetic field in the height direction. The conventional planar circuit proposed by Okoshi<sup>(1)</sup> consists of two parallel metal plates and a sandwiched dielectric as shown in Fig.1(a); here, the two metal plates work to confine the field in the height direction.

The surface-wave planar circuit considered here has a structure shown in Fig.1(b), where the dielectric constant is a function of height and the field is confined vertically in the form of a surface-wave. Typical structures of surface-wave planar circuits considered so far are summarized in Fig.2, where a

planar-type surface-wave transmission line and a surface-wave circular- or square-shaped resonator for each structure are also shown.

In order to construct a final surface-wave planar circuit which realizes the necessary characteristics or functions, a two dimensional side-wall must be prepared which works to confine the field laterally. The shape and kind of this two-dimensional side-wall are central problems of the synthesis of the planar circuit, but part of them have already been treated in planar circuit theory.

#### Formulation of Surface-Wave Planar Circuit Equation

The field distribution in the uniform dielectric planar structure shown in Fig.1(b) has two types of modes; that is TE and TM in the height direction. Field components for TE are  $E=(E_t, 0)$ ,  $H=(H_t, H_z)$  and that for TM are  $H=(H_t, 0)$ ,  $E=(E_t, E_z)$ . Since the dielectric constant is only the function of  $z$ , a separation of variable for field components is possible as shown by eqs. (A), (B) and (C) in table 1, which also gives the definition of planar voltage and current for each mode.  $f(z)$ ,  $g(z)$  and  $h(z)$  functions are defined by eqs. (G), (H) and (I) where eq. (G) is the eigen-value equation, and transverse propagation constant  $\beta_t$  (eq. (F)) is given as an eigen-value of this equation; suffix "n" means mode number of height mode; eq. (I) means orthogonality and normalization; upper and lower limit of integral of eq. (I) usually depends on the situation.

Then the surface-wave planar circuit equations which relate planar voltage and current are given by eqs. (D) and (E) where series impedance  $Z$  and shunt admittance  $Y$  are also defined. These relations are completely identical with the conventional planar circuit equations, given as follows

$$\text{grad}V = -ZJ \quad \text{div}J = -YV$$

#### Application of Surface-wave Planar Circuit Equations to Practical Problems

In order to show the usefulness of the surface-wave planar circuit equations, the covered image-line type is taken up and the following three simple examples are calculated based on these equations.

1. Scattering of an obliquely incident surface-wave of TE and TM type as shown in Fig.3(a).
2. Field distribution and propagation constant of covered image-line as shown in Fig.3(b).
3. Resonant field distribution and resonant frequency of circular-shaped resonator as shown in Fig.3(c).

The following considerations are taken into account in the above calculation;

- (1) Enough number of height mode is taken into account in order to satisfy the boundary condition (in the case of example 1, 50 TE and 50 TM modes are taken at each region).
- (2) Height of the cover  $h$  is taken as a parameter and is varied up to 16 times of the dielectric thickness  $d$ .

Part of the calculated results are shown in Fig.3 at the same time.

TABLE 1 Fundamental relation of surface-wave planar circuit.

	TE (H) MODE	TM (H) MODE	
Field component	$H_z(x, y, z) \equiv -V_n^H(x, y) \cdot g_n^H(z)$	$E_z(x, y, z) \equiv -V_n^E(x, y) \cdot g_n^E(z)$	A
	$E_t(x, y, z) \equiv [J_n^H(x, y) \times k] \cdot f_n^H(z)$	$H_t(x, y, z) \equiv [k \times J_n^E(x, y)] \cdot f_n^E(z)$	B
	$H_t(x, y, z) \equiv j \frac{1}{\eta_0} J_n^H(x, y) \cdot h_n^H(z)$	$E_t(x, y, z) \equiv j \eta_0 J_n^E(x, y) \cdot h_n^E(z)$	C
Planar circuit equations	$\text{grad}V_n^H = -Z_n^H J_n^H, \quad Z_n^H = j \frac{(\beta_n^H)^2}{\omega \mu} [S]$	$\text{grad}V_n^E = -Z_n^E J_n^E, \quad Z_n^E = j \frac{(\beta_n^E)^2}{\omega \epsilon_0} [\Omega]$	D
	$\text{div}J_n^H = -Y_n^H V_n^H, \quad Y_n^H = j \omega \mu [\Omega]$	$\text{div}J_n^E = -Y_n^E V_n^E, \quad Y_n^E = j \omega \epsilon_0 [S]$	E
	$\beta_t = \beta_n^H, \quad Z_n^H = \frac{\beta_n^H}{\omega \mu} [S], \quad Y_n^H = \frac{\omega \mu}{\beta_n^H} [\Omega]$	$\beta_t = \beta_n^E, \quad Z_n^E = \frac{\beta_n^E}{\omega \epsilon_0} [\Omega], \quad Y_n^E = \frac{\omega \epsilon_0}{\beta_n^E} [S]$	F
Basic surface-wave	$\frac{d^2 g_n^H}{dz^2} + [k_0^2 \epsilon_s(z) - (\beta_n^H)^2] g_n^H = 0$	$\frac{d}{dz} \left[ \frac{1}{\epsilon_s(z)} \frac{d}{dz} \{ \epsilon_s(z) g_n^E \} \right] + [k_0^2 \epsilon_s(z) - (\beta_n^E)^2] g_n^E = 0$	G
	$f_n^H = g_n^H, \quad h_n^H = \frac{1}{k_0} \frac{df_n^H}{dz}$	$f_n^E = \epsilon_s(z) g_n^E, \quad h_n^E = \frac{1}{k_0} \frac{1}{\epsilon_s(z)} \frac{df_n^E}{dz}$	H
	$\langle g_n^H, f_m^H \rangle \equiv \int g_n^H f_m^H dz = \delta_{mn}$	$\langle g_n^E, f_m^E \rangle \equiv \int g_n^E f_m^E dz = \delta_{mn}$	I

$k$ : unit vector toward height direction

$$k_0 = \omega \sqrt{\epsilon_0 \mu}, \quad \eta_0 = \sqrt{\mu / \epsilon_0}$$

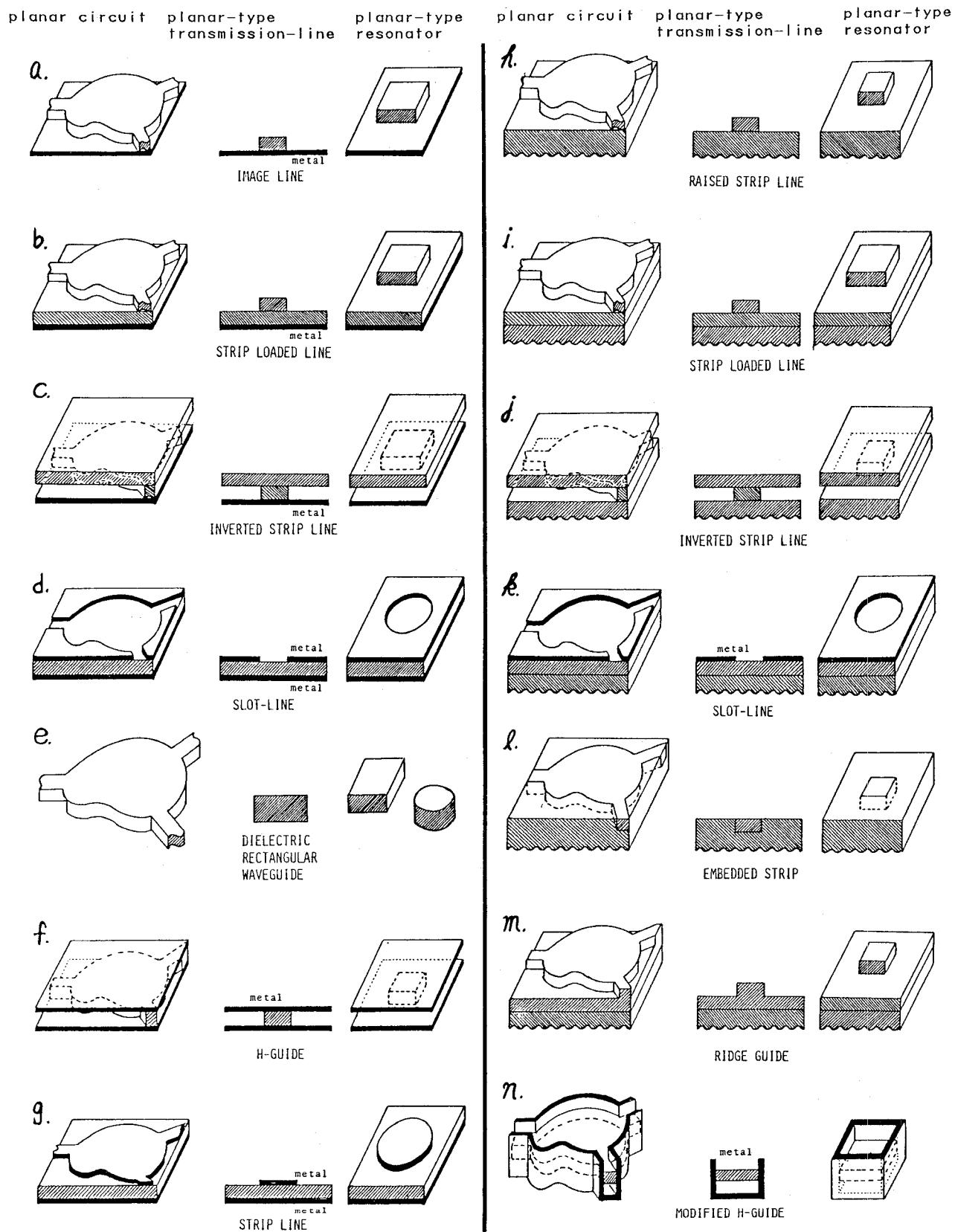


Fig.2 Typical structures of surface-wave planar circuit.

## Conclusion

The concept of a surface-wave planar circuit is proposed and typical structures of these circuits are summarized. Surface-wave planar circuit equations are formulated based on Maxwell's equations, where the existence of TE and TM mode is also pointed out. Finally, the concept of a surface-wave planar circuit is applied to practical problems; as examples covered image-line type surface-wave planar circuits are taken up and their circuit characteristics are calculated based on the surface-wave planar circuit equations.

## References

- (1) T. Okoshi, "Planar Circuits for microwaves and lightwaves." Springer-Verlag, 1985
- (2) P. P. Civalleri and S. Ridella, "Impedance and admittance matrices of distributed Three-layer N-ports." IEEE Trans. Circuit Theory, Vol. CT. 17, pp. 392-398, Aug. 1970.
- (3) Hsu Jui-Pang and T. Anada, "Planar circuit equation and its practical application to planar-type transmission-line circuit." in IEEE MTT Int. Symp. Dig. pp. 574-576, 1983. etc.
- (4) S. T. Peng and A. A. Oliner, "Guidance and leakage properties of a class of open dielectric waveguides: Part I." IEEE Trans. Vol. MTT-29, pp. 843-855, Sep. 1981.

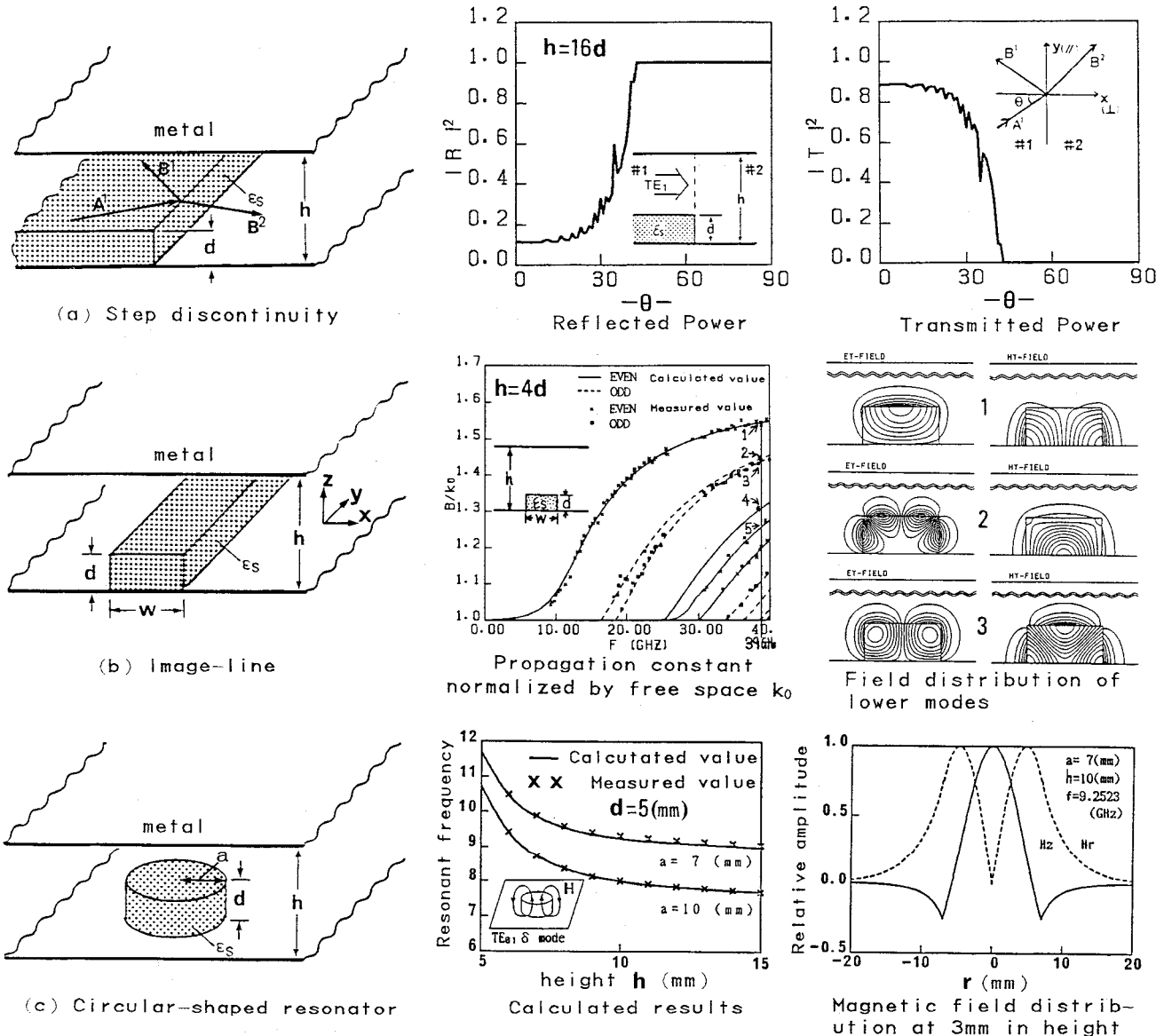


Fig.3 Simple example-covered image-line type surface-wave planar circuit-